CHOOSING COMPONENT VALUES FOR *L-C* RESONANT TANK CIRCUITS

Resonant 'tank' circuits consist of combinations of inductance (*L*) and capacitance (*C*) elements configured in such a manner that a single frequency — or more correctly a very narrow band of frequencies — is selected, while frequencies removed from the selected frequency are rejected.

By Joseph J. Carr

THE selection of a specific frequency occurs when the inductive reactance ($+X_L$) and capacitive reactance ($-X_C$) have equal magnitude, so therefore cancel each other. **Figure 1** shows two forms of *LC* resonant tank circuit, and a plot of impedance versus frequency. The parallel resonant form is shown in **Fig. 1a**, while the series resonant form is shown in **Fig. 1b**. The following rules apply to these forms (see **Fig. 1c**):

1. a parallel resonant circuit has a maximum impedance at its resonant frequency (F_0); and
2. a series resonant circuit has a minimum impedance at its resonant frequency (F_0).

In some cases, one of these types is clearly preferred over the other, but in other cases either could be used if it is used correctly. For example, in a wave trap — i.e., a circuit that prevents a particular frequency from passing — a **parallel resonant circuit in series with the signal line** will block its resonant frequency while passing all frequencies removed from resonance; a **series resonant circuit shunted across the signal path** will bypass its resonant frequency to common (or 'ground'), while allowing frequencies removed from resonance to pass.

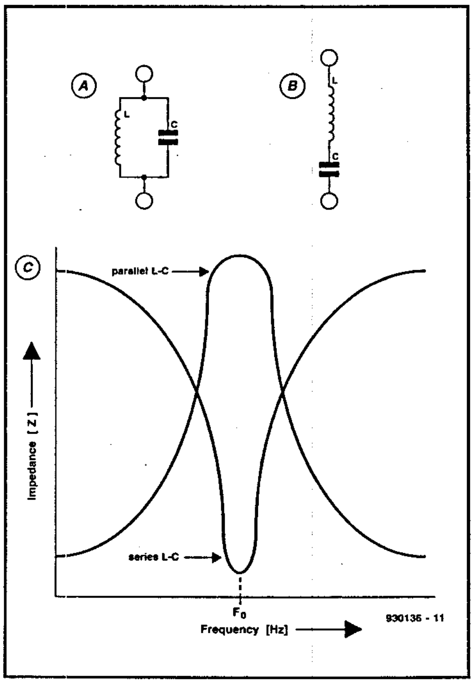

Fig. 1. a) Parallel resonant tank circuit; b) series resonant tank circuit; c) impedance-vs-frequency curves for series and parallel tank circuits.

LC resonant tank circuits are used to tune radio receivers; it is these circuits that select the station to be received, while rejecting others. A superheterodyne radio receiver (the most common type) is shown in simplified form in **Fig. 2**. According to the superhet principle, the radio frequency being received (F_{RF}) is converted to another frequency, called the **intermediate frequency** (F_{IF}), by being mixed with a **local oscillator** signal (F_{LO}) in a non-linear mixer stage. The output of the untamed mixer would be a collection of frequencies defined by:

$$F_{IF} = mF_{RF} \pm nF_{LO} \qquad (1)$$

Where m and n are either integers or zero. For the simplified case which is the subject of this article, only the first set of products (m=n=1) are considered, so the output spectrum will consist of F_{RF}, F_{LO}, $[F_{RF} - F_{LO}]$ (difference frequency), and $[F_{RF} + F_{LO}]$ (sum frequency). In older radios, for practical reasons the difference frequency was selected for F_{IF}; today, either sum or difference frequencies can be selected depending on the design of the radio.

There are several *LC* tank circuits present in this notional superhet radio. The antenna tank circuit (C1-L1) is found at the input of the RF amplifier stage, or if no RF amplifier is used it is at the input to the mixer stage. A second tank circuit (*L*2-*C*2), tuning the same range as *L*1-*C*1 is found at the output of the RF amplifier, or the input of the mixer. Another *LC* tank circuit (*L*3-*C*3) is used to tune the local oscillator; it is this tank circuit that sets the frequency that the radio will receive.

Additional tank circuits (only two shown) may be found in the IF amplifier section of the radio. These tank circuits will be fixed tuned to the IF frequency, which in common AM broadcast band (BCB) radio receivers is typically

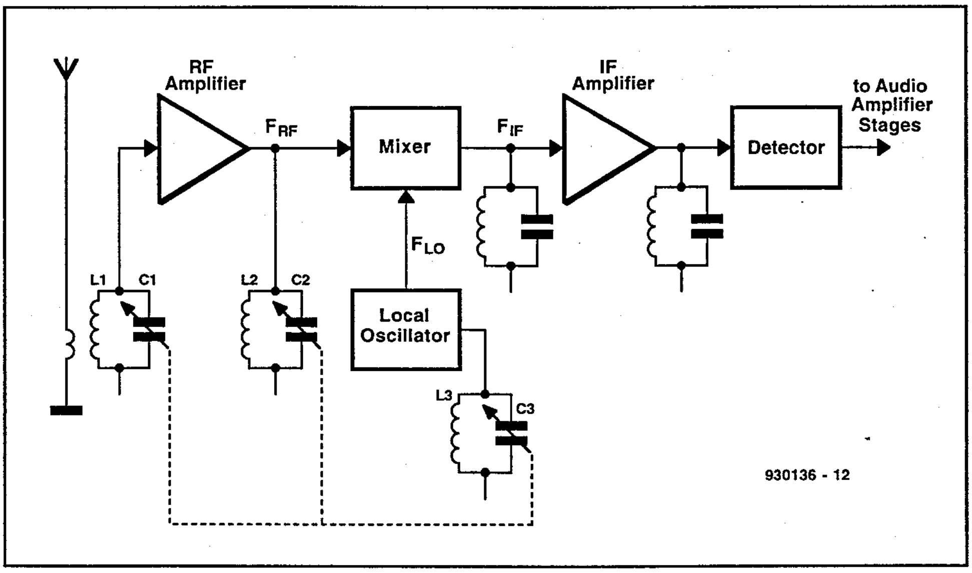

Fig. 2. *LC* tank circuits in a superheterodyne receiver.

450 kHz, 455 kHz, 460 kHz, or 470 kHz depending on the designer's choices (and sometimes country of origin). Other IF frequencies are also seen, but these are most common. FM broadcast receivers typically use a 10.7 MHz IF, while short-wave receivers might use a 1.65 MHz, 8.83 MHz, 9 MHz or an IF frequency above 30 MHz.

The tracking problem

On a radio that tunes the front-end with a single knob, which is almost all receivers today, the three capacitors (C_1, C_2 and C_3 in **Fig. 2**) are typically **ganged**, i.e., mounted on a single rotor shaft. These three tank circuits must **track** each other; i.e., when the RF amplifier is tuned to a certain radio signal frequency, the LO must produce a signal that is different from the RF frequency by the amount of the IF frequency. Perfect tracking is probably impossible, but the fact that your single knob tuned radio works is testimony to the fact that the tracking isn't too terrible.

The issue of tracking *LC* tank circuits for the AM broadcast band (BCB) receiver has not been a major problem for many years: the band limits are fixed over most of the world, and component manufacturers offer standard adjustable inductors and variable capacitors to tune the RF and LO frequencies. Indeed, some even offer three sets of coils: antenna, mixer input/RF amp output and LO. The reason why the antenna and mixer/RF coils are not the same, despite tuning the same frequency range, is that these locations see different distributed or 'stray' capacitances. In the U.S.A., it is standard practice to use a 10 to 365 pF capacitor and a 220 µH inductor for the 540 to 1600 kHz AM BCB. In some other countries, slightly different combinations are sometimes used: 320 pF, 380 pF, 440 pF, 500 pF and others are seen in catalogues (see, for instance, the Maplin Electronics catalogue. Address: P.O. Box 3, Rayleigh, Essex SS6 2BR, England).

Recently, however, two events coincided that caused me to examine the method of selecting capacitance and inductance values. First, I embarked on a design project to produce an AM DXers receiver that had outstanding performance characteristics. Second, the AM broadcast band was recently extended so that the upper limit is now 1700 kHz, rather than 1600 kHz. The new 540 to 1700 kHz band is not accommodated by the now-obsolete 'standard' values of inductance and capacitance. So I calculated new candidate values.

The RF amplifier/antenna tuner problem

In a typical RF tank circuit, the inductance is kept fixed (except for a small adjustment range that is used for

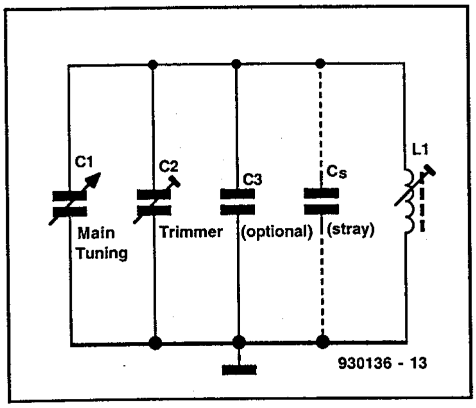

Fig. 3. Parallel resonant tuning circuit.

overcoming tolerance deviations) and the capacitance is varied across the range. **Figure 3** shows a typical tank circuit main tuning capacitor (C_1), trimmer capacitor (C_2) and a fixed capacitor (C_3) that is not always needed. The stray capacitance (C_s) includes the interwiring capacitance, the wiring to chassis capacitance, and the amplifier or oscillator device input capacitance. The frequency changes as the square root of the capacitance change. If F_1 is the minimum frequency in the range, and F_2 is the maximum frequency, then the relationship is:

$$\frac{F_2}{F_1} = \sqrt{\frac{C_{max}}{C_{min}}} \qquad (2)$$

or, in a rearranged form that some find more congenial:

$$\left(\frac{F_2}{F_1}\right)^2 = \frac{C_{max}}{C_{min}} \qquad (3)$$

In the case of the new AM receiver, I wanted an overlap of about 15 kHz at the bottom end of the band, and 10 kHz at the upper end, so needed a resonant tank circuit that would tune from 525 kHz to 1710 kHz. In addition, because variable capacitors are widely available in certain values based on the old standards, I wanted to use a 'standard' AM BCB variable capacitor. A 10 to 380 pF unit from a vendor was selected.

The minimum required capacitance, C_{min}, can be calculated from:

$$\left(\frac{F_2}{F_1}\right)^2 C_{min} = C_{min} + \Delta C \qquad (4)$$

Where:

F_1 is the minimum frequency tuned;
F_2 is the maximum frequency tuned;
C_{min} is the minimum required capacitance at F_2;
ΔC is the difference between C_{max} and C_{min}.

Example

Find the minimum capacitance needed to tune 1710 kHz when a 10 to 380 pF capacitor (ΔC= 380–10 pF = 370 pF) is used, and the minimum frequency is 525 kHz.

Solution:

$$\left(\frac{F_2}{F_1}\right)^2 C_{min} = C_{min} + \Delta C$$

$$\left(\frac{1710\,\text{kHz}}{525\,\text{kHz}}\right)^2 C_{min} = C_{min} + 370\,\text{pF}$$

$$10.609\, C_{min} = C_{min} + 370\,\text{pF}$$

$$C_{min} = 38.51\,\text{pF}$$

The maximum capacitance must be C_{min} + ΔC, or 38.51 + 370 pF = 408.51 pF. Because the tuning capacitor (C_1 in **Fig. 3**) does not have exactly this range, external capacitors must be used, and because the required value is higher than the normal value additional capacitors are added to the circuit in parallel to C_1. Indeed, because somewhat unpredictable 'stray' capacitances also exist in the circuit, the tuning capacitor values should be a little smaller than the required values in order to accommodate strays plus tolerances in the actual — versus published — values of the capacitors. In **Fig. 3**, the main tuning capacitor is C_1 (10 to 380 pF), C_2 is a small value trimmer capacitor used to compensate for discrepancies, C_3 is an optional capacitor that may be needed to increase the total capacitance, and C_s is the stray capacitance in the circuit.

The value of the stray capacitance can be quite high, especially if there are other capacitors in the circuit that are not directly used to select the frequency (e.g., in Colpitts and Clapp oscillators the feedback capacitors affect the *LC* tank circuit). In the circuit that I was using, however, the *LC* tank circuit is not affected by other capacitors. Only the wiring strays and the input capacitance of the RF amplifier or mixer stage need be accounted. From experience I apportioned 7 pF to C_s as a **trial** value.

The minimum capacitance calculated above was 38.51, there is a nominal 7 pF of stray capacitance, and the minimum available capacitance from C_1 is 10 pF. Therefore, the combined values of C_2 and C_3 must be (38.51–10–7 pF), or 21.5 pF. Because there is considerable reasonable doubt about the actual value of C_s, and because of tolerances in the manufacture of the main tuning variable capacitor (C_1), a wide range of capacitance for C_2+C_3 is preferred. It is noted from several catalogues that 21.5 pF is near the centre of the range of 45 pF and 50 pF trimmer capacitors. For example, one model lists its range as 6.8 pF to 50 pF, its centre point is only slightly removed from the actual desired capacitance. Thus, a 6.8 to 50 pF trimmer was selected, and C_3 is not used.

Selecting the inductance value for L_1 (**Fig. 3**) is a matter of picking the frequency and associated required capacitance at one end of the range, and calculating from the standard resonance equation solved for L:

$$L = \frac{10^6}{4\pi^2 F^2_{low} C_{max}} \quad \mu H \qquad (5)$$

$$L = \frac{10^6}{4\pi^2 (525{,}000)^2 (4.085\times10^{-10})}$$

$$L = 224.97 = 225\,\mu H$$

The RF amplifier input LC tank circuit and the RF amplifier output LC tank circuit are slightly different cases because the stray capacitances are somewhat different. In the example, I am assuming a JFET transistor RF amplifier, and it has an input capacitance of only a few picofarads. The output capacitance is not a critical issue in this specific case because I intend to use a 1 mH RF choke in order to prevent JFET oscillation. In the final receiver, the RF amplifier may be deleted altogether, and the LC tank circuit described above will drive a mixer input through a link coupling circuit.

The local oscillator (LO) problem

The local oscillator circuit must track the RF amplifier, and must also tune a frequency range that is different from the RF range by the amount of the IF frequency (455 kHz). In keeping with common practice I selected to place the LO frequency 455 kHz **above** the RF frequency. Thus, the LO must tune the range 980 kHz to 2,165 kHz.

There are three methods for making the local oscillator track with the RF amplifier frequency when single shaft tuning is desired: the **trimmer capacitor** method, the **padder capacitor** method, and the **different-value cut-plate capacitor** method.

The trimmer capacitor method was shown in **Fig. 3**, and is the same as the RF LC tank circuit. Using exactly the same method as before, but with a frequency ratio of (2165/980) to yield a capacitance ratio of $(2165/980)^2$ = 4.88:1, solves this problem. The results were a minimum capacitance of 95.36 pF, and a maximum capacitance of 465.36 pF. An inductance of 56.7 µH is needed to resonate these capacitances to the LO range.

There is always a problem associated with using the same identical capacitor for both RF and LO. It seems that there is just enough difference that tracking between them is always a bit off. **Figure 6** shows the ideal LO frequency and the calculated LO frequency. The difference between these two curves is the degree of non-tracking. The curves

```
100 'PROGRAM TO CALCULATE L-C TANK CIRCUIT VALUES
110 'A color monitor is needed for this program. If no color monitor is
120 'is used, then delete Line Nos. 140 and 150.
130   CLS:KEY OFF
140   SCREEN 9
150   GOSUB 1030:'Get graphic of circuit diagram
160 'Start of program
170     CLS:KEY OFF:SCREEN SCR:PI = 22/7
180 IF SCR > 0 THEN COLOR 1,7
190  LOCATE 6,15:PRINT "Calculate L-C tank circuit values from knowledge"
200  LOCATE  7,15:PRINT "of desired frequency range and variable capacitor"
210  LOCATE  8,15:PRINT "values (in picofards)."
220  LOCATE 10,15:PRINT "Enter MINIMUM frequency (F1)in kilohertz (KHz):";
230      INPUT F1$:' Obtain value of lowest frequency
240        F1 = VAL(F1$):' Test input of F1 to see if zero
250          IF F1 = 0 THEN BEEP
260          IF F1 = 0 THEN 220 ELSE 270
270  LOCATE 12,15:PRINT "Enter MAXIMUM frequency (F2) in kilohertz (KHz):";
280      INPUT F2$:' Obtain value of highest frequency
290        F2 = VAL(F2$):' Test input of F2 to see if zero
300          IF F2 = 0 THEN BEEP
310          IF F2 = 0 THEN 270
320            IF F2 = F1 THEN GOSUB 1440:'Test for incorrect freq. input
330            IF F1 > F2 THEN GOSUB 1440
340            IF F2 = F1 THEN 160 ELSE 350
350            IF F1 > F2 THEN 160 ELSE 360
360  LOCATE 14,15:PRINT "Select Capacitor Values..."
370  LOCATE 16,15:PRINT "Enter MINIMUM value of capacitor in picofarads (pF):";
380      INPUT CMIN$
390        CMIN = VAL(CMIN$)
400          IF CMIN = 0 THEN BEEP
410          IF CMIN = 0 THEN 360 ELSE 420
420  LOCATE 18,15:PRINT "Enter MAXIMUM value of capacitor in picofarads (pF):";
430      INPUT CMAX$
440        CMAX = VAL(CMAX$)
450          IF CMAX = 0 THEN BEEP
460          IF CMAX = 0 THEN 420 ELSE 470
470            IF CMIN = CMAX THEN GOSUB 1480:'Test for incorrect cap. input
480            IF CMIN > CMAX THEN GOSUB 1480:'and display error message
490            IF CMIN = CMAX THEN 360 ELSE 500
500            IF CMIN > CMAX THEN 360 ELSE 510
510  CLS:'Clear screen and print all entered values
520  LOCATE 6,15:PRINT "Minimum frequency (F1):";F1;" KHz"
530  LOCATE 7,15:PRINT "Maximum frequency (F2):";F2;" KHz"
540   LOCATE 9,15:PRINT "Minimum capacitance in C1: ";CMIN;" pF"
550   LOCATE 10,15:PRINT "Maximum capacitance in C1: ";CMAX;" pF"
560      FRATIO = F2/F1:'Calculate frequency ratio
570      CRATIO = FRATIO^2:'Calculate capacitance ratio
580      CDELTA = CMAX-CMIN
590   LOCATE 11,15:PRINT "Frequency ratio: ";
600     PRINT USING "##.###";FRATIO;:PRINT ":1"
610   LOCATE 12,15:PRINT "Required capacitance ratio: ";
620     PRINT USING "##.###";CRATIO;:PRINT ":1"
630   LOCATE 13,15:PRINT "Capacitance differential: ";
640     PRINT USING "####.##";CDELTA;:PRINT " pF"
650   XMIN = CDELTA/(CRATIO - 1):'Calculate min. total cap. needed
660   XMAX = CDELTA + XMIN:' Calculate max. total cap. needed
670   TRIMCAP = XMIN - CMIN
680     IF TRIMCAP < 0 THEN GOSUB 1580 ELSE 690
690     IF TRIMCAP < 0 THEN 160 ELSE 700
700   LOCATE 15,15:PRINT "Trimmer capacitance needed: ";
710   PRINT USING "####.##";TRIMCAP;:PRINT " pF"
720     LUH = 1/(4*PI^2*F1^2*XMAX*10^-12)
730   LOCATE 16,15:PRINT "Inductance needed: ";
740   PRINT USING "####.##";LUH;:PRINT " uH"
750   LOCATE 18,15:PRINT "Allocate a few pF for strays and then make up"
760   LOCATE 19,15:PRINT "remaining capacitance with either fixed or variable"
770   LOCATE 20,15:PRINT "trimmer capacitors."
780   LOCATE 22,15:GOSUB 1630:'Get press any key subroutine
790   CLS:'Clear screen for new message
800   LOCATE 10,15:PRINT "The trimmer capacitance will include all capacitances"
810   LOCATE 11,15:PRINT "seen by the inductor, and that could include a large"
820   LOCATE 12,15:PRINT "capacitance in other parts of the circuit. In certain"
830   LOCATE 13,15:PRINT "oscillator circuits, especially Colpitts and Clapp"
840   LOCATE 14,15:PRINT "oscillators, the other capacitances are large." .
850   LOCATE 15,15:PRINT "Check a good oscillator design handbook for further"
860   LOCATE 16,15:PRINT "information."
870   LOCATE 18,15:GOSUB 1630
880   CLS:LOCATE 12,15:PRINT "(D)o Another Problem?"
890   LOCATE 13,15:PRINT "(E)nd Program?"
900   CHOICE$ = INPUT$(1)
910   IF CHOICE$ = "D" THEN CHOICE = 1
920   IF CHOICE$ = "d" THEN CHOICE = 1
930   IF CHOICE$ = "E" THEN CHOICE = 2
940   IF CHOICE$ = "e" THEN CHOICE = 2
950      IF CHOICE < 1 THEN 880 ELSE 960
960      IF CHOICE > 2 THEN 880 ELSE 970
970      IF INT(CHOICE) = CHOICE THEN 980 ELSE 880
980   ON CHOICE GOTO 160,990
990 CLS:LOCATE 12,20:PRINT "Program Ended...Goodbye"
1000  TIMELOOP=TIMER:WHILE TIMER < TIMELOOP + 2:WEND
1010 CLS
1020 END
1030 'Subroutine to draw circuit picture on screen
1040 CLS:LINE (450,100)-(100,100):'Draw connecting rails
1050 LINE (450,200)-(100,200)
1060 LINE (175,180)-(165,120),,B:'Draw inductor L1
1070 LINE (170,200)-(170,180)
1080 LINE (170,120)-(170,100)
1090 LINE (175,180-ZZ)-(165,180-ZZ)
1100 ZZ = ZZ + 5
1110 IF ZZ =  60 GOTO 1130
1120 GOTO 1090
1130 LINE (250,200)-(250,153):'Draw C1 (main tuning capacitor)
1140 LINE (250,147)-(250,100)
1150 LINE (265,153)-(235,153)
1160 LINE (265,147)-(235,147)
1170 LINE (265,165)-(235,135)
1180 LINE (350,200)-(350,153):'Draw C2 (trimmer capacitor)
1190 LINE (350,147)-(350,100)
1200 LINE (365,153)-(335,153)
1210 LINE (365,147)-(335,147)
1220 LINE (365,165)-(335,135)
1230 LINE (450,200)-(450,153):'Draw stray capacitances
1240 LINE (450,147)-(450,100)
1250 LINE (465,153)-(435,153)
1260 LINE (465,147)-(435,147)
1270 LINE (102,102)-(98,98),1,B
1280 LINE (102,202)-(98,198),1,B
1290 LINE (252,202)-(248,198),1,BF
1300 LINE (252,102)-(248,98),1,BF
1310 LINE (352,202)-(348,198),1,BF
1320 LINE (352,102)-(348,98),1,BF
1330 LINE (172,202)-(168,198),1,BF
1340 LINE (172,102)-(168,98),1,BF
1350 LOCATE 11,19:PRINT "L1":'Add alphabetic labels
1360 LOCATE 11,27:PRINT "C1"
1370 LOCATE 11,40:PRINT "C2"
1380 LOCATE 11,49:PRINT "Stray"
1390 LOCATE 16,15:PRINT "C1 is main tuning capacitor"
1400 LOCATE 17,15:PRINT "C2 is trimmer + fixed capacitors"
1410 LOCATE 18,15:PRINT "Stray is wiring capacitance plus other"
1420 LOCATE 19,15:PRINT "capacitors in the circuit."
1430 LOCATE 21,15:GOSUB 1630:CLS:RETURN:'End of Subroutine
1440 ' Error Message Subroutine (Incorrect Frequency Input)
1450   BEEP:LOCATE 17,15:PRINT "ERROR!  F2 must be greater than F1"
1460   LOCATE 19,15:GOSUB 1630:' Go get press any key subroutine
1470 RETURN:'End of Subroutine
1480 'Error Message Subroutine (Incorrect Capacitance Input)
1490  BEEP:LOCATE 20,15:PRINT "ERROR! Cmax must be greater than Cmin"
1500   LOCATE 22,15:GOSUB 1630:' Go get press any key subroutine
1510   GOSUB 1530:'Clear screen and reprint F1 and F2
1520  RETURN:' End of Subroutine
1530 ' Subroutine to clear screen and reprint values of F1 and F2
1540  CLS
1550  LOCATE 6,15:PRINT "Minimum frequency (F1):";F1;" KHz";
1560  LOCATE 7,15:PRINT "Maximum frequency (F2):";F2;" KHz"
1570 RETURN:'End of Subroutine
1580 'Subroutine for Too Small Trimmer Capacitance
1590  CLS
1600  LOCATE 12,15:PRINT "ERROR!!! Not a viable combination...try again"
1610  LOCATE 14,15:GOSUB 1630:'Go get press any key subroutine
1620  RETURN
1630 ' Press Any Key... Subroutine
1640 PRINT "Press Any Key To Continue..."
1650 A$ = INKEY$:IF A$ = "" THEN 1650 ELSE 1660
1660 RETURN:'End of Subroutine
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Fig. 4. BASIC program to calculate RF tank circuit component values.

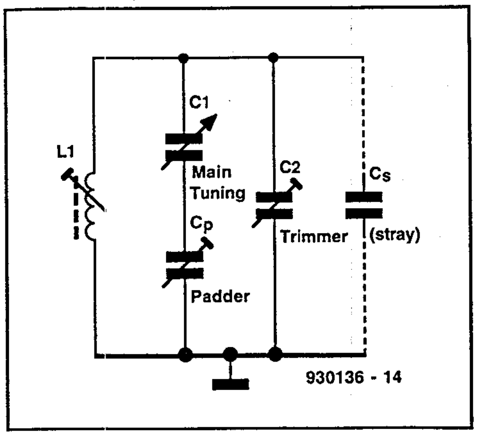

Fig. 5. Parallel resonant tank circuit using the padder capacitor method.

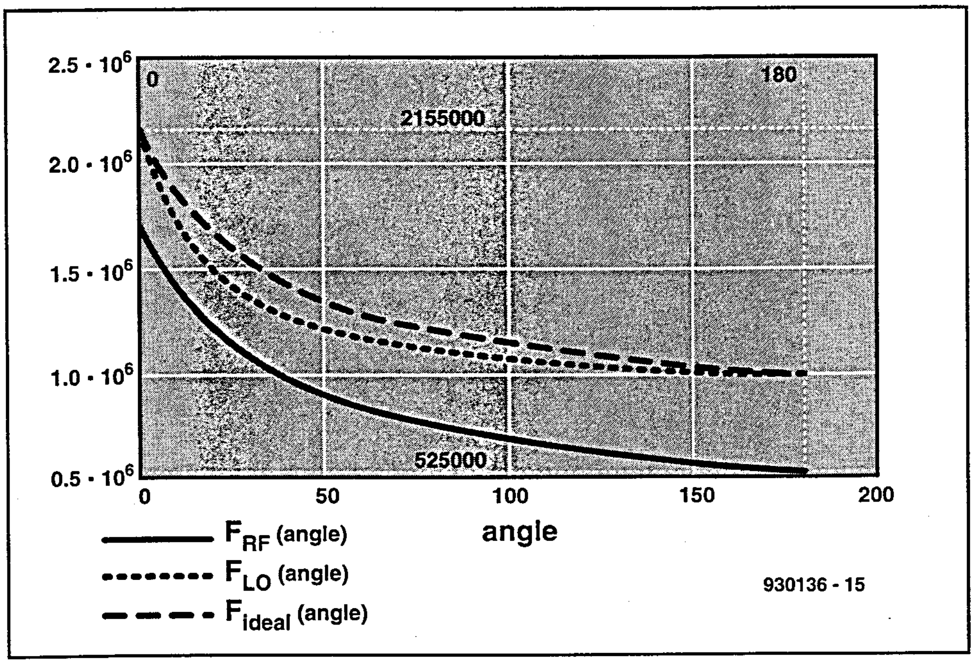

Fig. 6. MathCAD 3.1 plot of RF, LO (ideal) and LO (actual) tuning as a function of capacitor shaft angle.

overlap at the ends, but are awful in the middle. There are two cures for this problem. First, use a **padder capacitor** in series with the main tuning capacitor (**Fig. 5**). Second, use a **different-value cut-plate capacitor**.

Figure 5 shows the use of a padder capacitor (C_p) to change the range of the LO section of the variable capacitor. This method is used when both sections of the variable capacitor are identical. Once the reduced capacitance values of the $C1/C_p$ combination are determined the procedure is identical to above. But first, we have to calculate the value of the padder capacitor and the resultant range of the $C1/C_p$ combination. The padder value is found from:

$$\frac{C1_{max}\,C_p}{C1_{max}+C_p}=\left(\frac{F2}{F1}\right)^2\left(\frac{C1_{min}\,C_p}{C1_{min}+C_p}\right) \quad (6)$$

...and solving for C_p. For the values of the selected main tuning capacitor and LO:

$$\frac{(380\,\text{pF})\,C_p}{(380+C_p)\,\text{pF}}=(4.88)\left(\frac{(10\text{pF})\,C_p}{(10+C_p\,\text{pF})}\right)$$

Solving for C_p by the least common denominator method (crude, but it works) yields a padder capacitance of 44.52 pF. The series combination of 44.52 pF and a 10 to 380 pF variable yields a range of 8.2 pF to 39.85 pF. An inductance of 661.85 µH is needed for this capacitance to resonate over 980 kHz to 2,165 kHz.

A practical solution to the tracking problem that comes close to the ideal is to use a **cut-plate** capacitor. These variable capacitors have at least two sections, one each for RF and LO tuning. The shape of the capacitor plates are especially cut to a shape that permits a constant change of **frequency** for every degree of shaft rotation. With these capacitors, when well done, it is possible to produce three-point tracking, or better.■